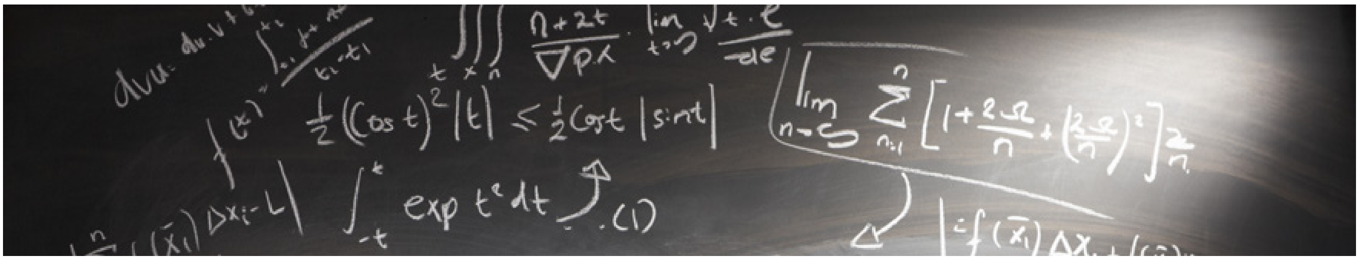
The Sauerbrey relation



Resonance in an AT-cut piezoelectric quartz crystal will occur when the thickness of the crystal is an odd integer of half wavelengths of the induced wave, and the oscillation will have its anti-nodes at each surface. The resonant frequency can then be stated as:

$$f = n\frac{v_q}{2t_q} = n \cdot f_0 \qquad (1)$$

f – resonance frequency, s^{-1}

f_0 – the fundamental resonance frequency, s^{-1}

n – harmonic number

v_q – wave velocity in quartz plate[1], m s^{-1}

t_q – thickness of quartz plate, m

ρ_q – density of quartz[2], kg/m^3

M – total added mass, kg

A – Active area, m^2

m_q – Areal mass, kg·m^{-2}

The fundamental resonant frequency, where n=1 can be expressed as:

$$f_0 = n\frac{v_q}{2t_q} \qquad (2)$$

The mass per area can be expressed as the product of thickness and density, if evenly distributed:

$$\frac{M}{A} = m_q = t_q \cdot \rho_q \quad [m \cdot \frac{kg}{m^3} = \frac{kg}{m^2}] \quad (3)$$

by inserting (3) into equation (2) and differentiating the expression, it becomes:

$$df = -\frac{f}{m_q}dm_q \qquad (4)$$

Change in added mass *m* on the crystal can be treated as en equivalent of the crystal itself, provided that the added mass is:

(i) small compared to the crystal

(ii) rigidly adsorbed with no slip or deformation imposed by the oscillating surface

(iii) evenly distributed

By letting $d \rightarrow \Delta$, and using eqations 1, 2 and 3 to replace m_q and f in equation (4) we get:

$$df = \Delta f = -\frac{f}{t_q \cdot \rho_q}\Delta m = -n \cdot \frac{2f_0^2}{v_q \cdot \rho_q}\Delta m = -n \cdot \frac{1}{C}\Delta m \qquad (5)$$

By performing unit analysis of equation (5), the unit of the constant C equals:

$$C = \frac{v_q \cdot \rho_q}{2f_0^2} \quad [\frac{\frac{m}{s}\cdot\frac{kg}{m^3}}{\frac{1}{s^2}} = \frac{kg \cdot s}{m^2}] \qquad (6)$$

When equation (5) is used to calculate mass changes on a crystal, it can be rewritten on the following form:

$$\Delta m = -C \cdot \frac{\Delta f}{n} \qquad (7)$$

in which case *m* is the areal mass difference.

Active area

By using equation (7) for estimations of added mass, it is not necessary to know the active area as long as the assumption (i, iii, iii) are fulfilled.

References

Original work by Sauerbrey: Sauerbrey, Z. Phys. **1959**, 155, 206-222.

For a modern explanation (in English): Höök, F. Development of a novel QCM technique for protein adsorption studies, **1997,** Chalmers University of Technology, Department of Biochemistry & Biophysics and Department of Applied Physics, PhD Thesis.

Footnote:
[1]Wave velocity in an AT-cut quartz plate, v_q = 3340m/s
[2]Density of AT-cut quartz ρ_q = 2650 kg/m^3

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